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ON THE DETERMINATION OF THE DYNAMICAL OBLATENESS OF THE SUN AND OF THE COEFFICIENTS β AND γ OF THE GENERALIZED TENSOR OF ROBERTSON

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ON THE DETERMINATION OF THE DYNAMICAL OBLATENESS OF THE SUN AND OF THE COEFFICIENTS β AND γ OF THE GENERALIZED TENSOR OF ROBERTSON

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ABSTRACT. The motion of a probe on an heliocentric orbit under the influence of the relativistic effects and the oblateness of the Sun is integrated up to second order terms.

The coefficient γ could be obtained with a great accuracy (10^{-3} to 10^{-4}) by the observation of laser beams coming from the probe and passing close to the Sun.

The coefficients β and J_{20} could be deduced with limited accuracy (1 to 20% for β, 10^{-6} to 4. 10^{-5} for J_{20}) from the observation of the perturbations of the orbit. The non-Newtonian advance of the perihelion of Mercury is known with an accuracy of 1% and results in a very helpful relationship between β and J_{20}. On the contrary, the motion of the node of Mercury, which leads to a very small value of J_{20} if the solar equator is the apparent one, is contested by many scientists because of the inaccurate definition of the solar equator.

Some possible experiments are suggested and compared with the Viking project (Mars lander).

The second order effects are too small to be useful.

I. INTRODUCTION

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The various theories of general relativity and gravitation constructed from Einstein's theory essentially differ by their structure of space-time

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*Numbers in the margin indicate pagination in the original foreign text.

with respect to heavy volumes. The metric tensor (for example, formulated in the Robertson form) around a body having spherical symmetry remains independent of time and has spherical symmetry, but varies as a function of radial distance in different ways.

In order to compare these theories it would be appropriate to carry out experiments for measuring the coefficients of the metric tensor, i.e., to primarily measure the coefficients β and γ of the Robertson tensor. A first relationship is obtained by the advance of the perihelion of Mercury. However, this relationship, in which the error is on the order of 1%, introduces the dynamic flattening of the Sun, i.e., the J_{20} coefficient of this heavenly body.

The coefficient γ only enters in the trajectory lengthening of light rays passing near the Sun. The experiments of Shapiro (reflections of electromagnetic waves at a planet) give them with an accuracy of 10 to 20%, and it is possible to considerably improve this accuracy by using a laser beam transmitted to a probe passing around the Sun. The accuracy of laser distance measurements will be assumed to be equal to three meters.

The coefficients β and J_{20} can be derived from the perturbations of the orbit with a much better accuracy.

II. MOTION OF A SPACE PROBE AROUND AN ATTRACTING CENTER HAVING SPHERICAL SYMMETRY ACCORDING TO THE THEORY OF GENERAL RELATIVITY

Let us use the speed of light as the velocity unit.

The most general static and spherical space-time tensor can be expressed in the form of Robertson [1]:

$$ds^2 = f(\rho)\, dt^2 - g(\rho)\,(dx^2 + dy^2 + dz^2)$$

where ρ is the radial distance $= \sqrt{x^2 + y^2 + z^2}$. f and g are two functions of ρ which are equal to 1 at infinity:

$$f(\rho) = 1 - \frac{2\alpha m}{\rho} + \frac{2\beta m^2}{\rho^2} + \dots$$

$$g(\rho) = 1 + \frac{2\gamma m}{\rho} + \dots$$

αm is a length equal to $\frac{GM}{C^2}$ ($= 1{,}5$ km in the case of the Sun) because the motion is zero order Newtonian. We may always set $\alpha = 1$.

The theory of Einstein leads to $\beta = \gamma = 1$, but other theories (Jordan, Thiry, Brans-Dicke) lead to smaller values of γ.

In order to test these theories, let us study the motion given by the Robertson tensor.

We will use the Schwarzschild form in which the coefficient of circumferential differentials is equal to 1:

Let us set:

r = new radial distance = $\rho \sqrt{g(\rho)} = \rho + \gamma m + \dots$

φ = latitude
L = longitude

ds^2 becomes:

$$ds^2 = F(r)\,dt^2 - G(r)\,dr^2 - r^2\,(d\varphi^2 + \cos^2\varphi\,.\,dL^2)$$

$$\text{with } F(r) = f(\rho) = 1 - \frac{2m}{r} + \frac{2m^2}{r^2}(\beta - \gamma) + \dots$$

$$G(r) = \left(1 + \frac{\rho}{2g}\frac{dg}{d\rho}\right)^{-2} = 1 + \frac{2\gamma m}{r} + \dots$$

The motion is planar in the plane $\phi = 0$; the equations of motion become:

$r^2 \frac{dL}{ds} = h$ integral of the kinetic moment.

$F(r).\frac{dt}{ds} = k$ = constant

$$G(r).\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{dL}{ds}\right)^2 + 1 - F(r)\left(\frac{dt}{ds}\right)^2 = 0:$$

expression of the ds^2.

The parameter t is a "regularized time" or "cosmic time" (time at a large distance and zero velocity); the parameter s is the proper time of the probe.

These equations can be easily integrated to the first order using an auxiliary Keplerian motion.

Let us set a priori:

$$p = \frac{h^2}{m} + m\left(2\beta - 4 - \frac{5\gamma}{2}\right) + \frac{h^2}{m}(1-k^2)\left(1+\frac{\gamma}{2}\right)$$

$$e^2 = k^4 + \frac{h^2}{m^2}(k^2-1) - \frac{\gamma m^2}{h^2} + (1-k^2)(6+4\gamma-2\beta) - \frac{h^2}{m^2}(1-k^2)^2(2+\gamma)$$

and let us consider the Keplerian orbit having eccentricity e and with a parameter p around this same attraction center (we will use the usual parameters n, a, b, p, e, v, E, M, and the graviational constant μ(= n^2a^3) is equal to m here).

We therefore obtain the following as a function of p, e, and a $\left(=\frac{p}{1-e^2}\right)$:

$$h^2 = mp + m^2\left[3 + 2\gamma - 2\beta + e^2 + \frac{\gamma e^2}{2}\right] + \text{order}\frac{m^3}{p}$$

$$k^2 = 1 - \frac{m}{a} + \frac{m^2}{a^2} + \frac{\gamma m^2 e^2(1+e^2)}{2p^2} + \text{order}\frac{m^3}{p^3}$$

The formulas which relate the constants are obviously selected in such a way that the equations of motion will be simplified as much as possible. These can be easily expressed as a function of the true anomaly v:

$$\frac{1}{r} = 1 + e\cos v - \frac{\gamma m e^2}{2p^2}\cos 2v$$

$$\varphi = 0 \text{ by hypothesis}$$

$$L = L_0 + v\left[1 + \frac{m}{p}(2+2\gamma-\beta)\right]$$

$$s = s_0 + \frac{M}{n}\left[1 + \frac{m}{2a} + \frac{\gamma m}{p}\left(1 - \frac{e^2}{4} + \frac{3e^4}{2(1-e^2)}\right)\right]$$

$$+\frac{\gamma me^2}{np\,(1-e^2)}\left[\left(\frac{3e^3}{2}-2e\right)\sin E+\frac{2-e^2}{4}\sin 2E\right.$$

$$t=t_0+(s-s_0)\left(1+\frac{3m}{2a}\right)+\frac{2me\,\sin E}{na}$$

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that is:

$$t=t_0+\frac{M}{n}\left[1+\frac{2m}{a}+\frac{\gamma m}{p}\left(1-\frac{e^2}{4}+\frac{3e^4}{2\,(1-e^2)}\right)\right]$$
$$+\frac{2me\sin E}{na}+\frac{\gamma me^2}{np\,(1-e^2)}\left[\left(\frac{3e^3}{2}-2e\right)\sin E+\frac{2-e^2}{4}\sin 2E\right]$$

and, in the usual way

$$tg\frac{E}{2}=tg\frac{v}{2}\cdot\sqrt{\frac{1-e}{1+e}}\quad\text{and}\quad M=E-e\sin E;$$

$\frac{M}{n}$ is the proper time of Keplerian orbit, and L_0, S_0, t_0 are three integration constants.

These equations give the position, the proper time, and the "cosmic time" as a function of v, that is, all the motion parameters. They are valid (to first order) for any eccentricity and, for spherical symmetry, for any inclination.

Remarks:

1. β only appears in the expression for L.

2. The only visible secular effect is the advance of the perihelion

$$\left(\text{angular velocity}:\frac{nm}{\rho}(2+2\gamma-\beta)\right)$$

3. The second order term results in lengths on the order of $\frac{m^2}{p}$, and never exceeds 4 mm in the case of the Sun.

III. MOTION OF A SPACE PROBE UNDER THE INFLUENCE OF THE SUN'S FLATTENING

Let the usual potential of a flattened body be given by the following expression (R is the equatorial radius of the body):

$$U = -\frac{m}{r}\left[1 + J_{20} \cdot \frac{R^2}{r^2}\frac{3\sin^2\varphi - 1}{2} + \dots\right]$$

and let us neglect the terms after J_{20}.

The motion of a free probe in this potential can be expressed, to terms up to second order, by means of an auxiliary Keplerian orbit O.

The elements of O are, as usual: n, a, b, p, e, i, Ω, ω, v, E, M with the following relationships:

$$b = a\sqrt{1-e^2},\ p = a(1-e^2),\ n^2 a^3 = m,$$
$$tg\frac{E}{2} = tg\frac{v}{2}\cdot\sqrt{\frac{1-e}{1+e}},\ M = E - e\sin E.$$

The orbit O rotates around the polar axis and around its normal axis at angular rates which are proportional to those of the true anomaly v (this considerably simplifies the equations of motion):

$$\Omega = \text{longitude of the node} = \Omega_0 + v\cdot\frac{3J_{20}R^2}{2p^2}\cos i$$

ω = angle of the node at the perihelion (measured in the direction of motion)

$$= \omega_0 + v\cdot\frac{3J_{20}R^2}{4p^2}(1 - 5\cos^2 i)$$

Ω_0 and ω_0 are two integration constants.

Let us denote by r_1, ϕ_1, L_1 the coordinates of the point moving along the rotating orbit O:

$$r_1 = \frac{p}{1 + e\cos v} = a(1 - e\cos E)$$

$$\varphi_1 = \text{Arc sin}\left[\sin i.\sin(v+\omega)\right] \Big| \text{thus}: tg\,\varphi_1 = tg\,i.\sin(L_1 - \Omega)$$

$$L_1 = \Omega + \text{arc}\,tg\left[\cos i.tg(v+\omega)\right] \Big| \text{and: } \cos(v+\omega) = \cos\varphi_1.\cos(L_1 - \Omega).$$

The motion in the potential U can therefore be expressed, up to second order terms [2], as a function of the true anomaly v of the orbit O (v is a true descriptive parameter) with six appropriate integration constants a, e, i, Ω_0, ω_0 and t_0.

We obtain

$$r = r_1 - J_{20}R^2\left[\frac{r_1^2}{p^3}\left(\frac{e^2}{2}\cos 2i\cos 2v - \frac{e^3}{16}\sin^2 i\cos 3v\right) + \frac{\cos^2\varphi_1}{2r_1}\right]$$

$$\varphi = \varphi_1 + J_{20}R^2\left[\frac{\sin 2\varphi_1}{4r_1^2} - \frac{\sin^3 i.\sin^2(v+\omega).\cos(L_1-\Omega)}{8b^2}\right]$$

$$L = L_1 + J_{20}R^2\left[\frac{e\cos i\sin v\,(4+e\cos v)}{2p^2} - \frac{\sin^2 i\sin 2(L_1-\Omega)}{8b^2}\right]$$

$$t = \text{time} = t_o + \frac{M}{n}\left[1 - \frac{3J_{20}R^2}{p^2}\left(\frac{\cos^2 i}{2} - \frac{e^2}{4} + \frac{e^4}{1-e^2}\left(\frac{1}{2} - \frac{15\sin^2 i}{16}\right)\right)\right]$$

$$+ \frac{J_{20}R^2}{4nab}\sin^2 i\sin 2(v+\omega)$$

$$- \frac{J_{20}R^2 e}{np^2(1-e^2)}\left(\begin{array}{l}\sin E\left(\frac{3e^4}{2} - 2e^2\right) + \sin 2E\left(\frac{e}{2} - \frac{e^3}{4}\right)\\ + \sin^2 i\sin E\left[1 - \frac{19}{8}e^2 + \frac{21}{16}e^4\right]\\ + \sin^2 i\sin 2E\left[\frac{e}{8} - \frac{3e^3}{32}\right]\end{array}\right).$$

These equations give the position in time as a function of the auxiliary true anomaly v. They are valid, to second order, for any inclination and any eccentricity.

IV. ESTIMATION OF β, γ, AND J_{20} BY THE STUDY OF THE MOTION OF A SOLAR PROBE

1. Measurements.

Radio-electric and laser measurements do not directly give the distance d (Figure 1). Instead, because of the curvature of space, they result in a length L equal to:

$$L = d - m(1+\gamma)\,\mathrm{Log}\left(\mathrm{tg}\frac{A}{2}\,\mathrm{tg}\frac{B}{2}\right) - \gamma m(\cos A + \cos B) + \text{order}\,\frac{m^2}{r_T \sin A}.$$

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This results in a convenient way of measuring γ to great accuracy when A and B vary near zero (superior conjunction).

Remarks: L does not depend on the radial parameter which we are using, and it has the same value if we utilize ρ instead of r.

2. Estimation of γ.

Let us use a drag-free satellite and assume that the long period perturbations are of order Γ, for example, 10^{-11} or 10^{13} m/s^2.

On the other hand, let us assume that the length L

$$\left(L = d - m(1+\gamma)\,\mathrm{Log}\left(\mathrm{tg}\frac{A}{2}\,\mathrm{tg}\frac{B}{2}\right) - \gamma m(\cos A + \cos B)\right)$$

is measured with an accuracy of three meters (laser measurements).

At an acceleration of 10^{-11} m/s^{2*}, meters are obtained in nine days. Therefore let us measure L over 18 days. That is, nine days before and nine

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*Translator's Note: Number omitted here in foreign text.

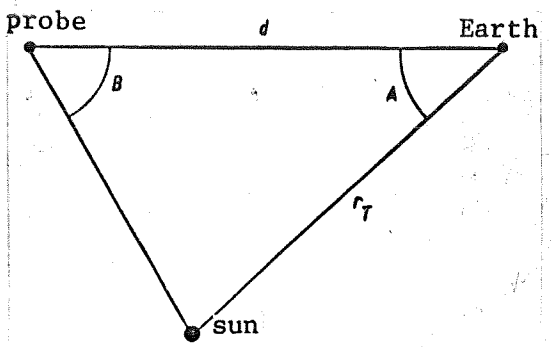

Figure 1. Measurements

days after the passage of the probe behind the Sun. The measurement depends considerably on the angular rates $\frac{dA}{dt}$ and $\frac{dB}{dt}$ at the time of conjunction.

Let us therefore consider the following six orbits tangential to those of the Earth near the point of departure:

Period	Semi-Major Axis	Perihelion Distance	Aphelion Distance	Eccentricity	$\frac{dA}{dt}$ (in °/day at the time of conjunction)	$\frac{dB}{dt}$
6 mos.	0,630 U.A.*	0,260 U.A.	1 U.A.	0,587	0,177	0,177
8 mos.	0,763 U.A.	0,526 U.A.	1 U.A.	0,310	0,681	1,293
9 mos.	0,825 U.A.	0,651 U.A.	1 U.A.	0,211	0,055	0,055
16 mos.	1,211 U.A.	1 U.A.	1,423 U.A.	0,174	0,269	0,189
18 mos.	1,310 U.A.	1 U.A.	1,621 U.A.	0,237	0,055	0,055
24 mos.	1,587 U.A.	1 U.A.	2,175 U.A.	0,370	0,508	0,234

*U.A. = A.U.

The best orbit is therefore the eight-month orbit. In ± nine days A varies from + 6.1° to -6.1° and B varies from + 11.6° to -11.6°. The angular radius of the sun is 0.25°. We can use variations of A between 6.1° and 0.25° or 0.3°. This gives a variation of $\gamma m \operatorname{Log}\left(tg\frac{A}{2}\,tg\frac{B}{2}\right)$ of γ.7.5 km or γ.9 km. The accuracy of the measurement of γ is of order $\frac{3m}{7,5 \text{ à } 9 \text{ km}}$, that is, 3 to 4.10^{-4}.

The accuracy is 5.10^{-4} for a 24-month orbit, 6.10^{-4} for a 16-month orbit, and is much less for the three others.

These accuracies become 2.10^{-4} for any orbit if the unknown long-period perturbations Γ are reduced to 10^{-13} m/s^2.

3. Estimation of β and J_{20}.

If the orbit of the probe has a large inclination (with respect to the solar equator) and a large eccentricity, β and J_{20} can be derived from the perturbations to this orbit.

The perturbations are of course additive, to second order, and, for example, in the case i = 0 the motion is given by:

$$r = r_1 + \frac{\gamma m e^2 r_1^2}{2p^2}\cos 2v - \frac{J_{20}R^2}{2}\left[\frac{e^2 r_1^2}{p^3}\cos 2v + \frac{1}{r_1}\right];$$

with $r_1 = \dfrac{p}{1 + e\cos v}$

$\phi = 0$, because i = 0

$$L = L_0 + v\left[1 + \frac{m}{p}(2 + 2\gamma - 3) - \frac{3J_{20}R^2}{2p^2}\right] + J_{20}\frac{R^2}{p^2}\left(2e\sin v + \frac{e^2}{4}\sin 2v\right)$$

$$t = t_0 + \frac{M}{n}\left[1 + \frac{2m}{a} + \frac{\gamma m}{p}\left(1 - \frac{e^2}{4} + \frac{3e^4}{2(1-e^2)}\right) - \frac{3J_{20}R^2}{4p^2}\left(2 - e^2 - \frac{7e^4}{4(1-e^2)}\right)\right] + \frac{2me\sin E}{na} + \frac{e^2}{1-e^2}\left[\frac{\gamma m}{np} - \frac{J_{20}R^2}{np^2}\right]\left[\sin E\left(\frac{3e^3}{2} - 2e\right) + \sin 2E\,\frac{2-e^2}{4}\right]$$

s = proper time of the probe

$$= s_0 + (t - t_0)\left(1 - \frac{3m}{2a}\right) - \frac{2me\sin E}{na}$$

L_0, t_0, s_0 are three integrated constants.

Of course it is necessary to keep track of the errors in these constants of integration, the orbital parameter and the gravitational constant of the Sun m. The greatest lengths which can be measured in a revolution must be averaged and are given by (if only distance measurements are available):

(1) L_1, connected to the perihelion advance:

$$L_1 = \pi ae\left[\frac{3 J_{20} R^2}{2 p^2}(2 - 3\sin^2 i) + \frac{2\beta m}{p}\right].$$

(2) L_2, connected to the periodic variations if the eccentricity is large and to the rotation of the orbital plane if the inclination is large: /61

$$L_2 = \frac{J_{20} R^2}{2p} \cdot \max\left[e^2; \frac{6\pi \sin i \sin\alpha\, |\cos i|}{1 - e}\right]$$

i is the inclination with respect to the solar equator, and α is the inclination with respect to the ecliptic.

L_1 and the second term of L_2 increase in proportion to the number of revolutions, but since it has the same degree of error as the unknown long period perturbations Γ, the accuracy of the measurements does not increase with time.

Γ results in an orbital displacement on the order of $\frac{2\pi p^3 \Gamma}{mc^2}$ per revolution (c is the speed of light). The accuracy of the measurements is thus approximately:

For J_{20}

$$\delta J_{20} = \frac{\frac{2\pi p^3 \Gamma}{mc^2}}{L_2 / J_{20}}$$

or:

$$\delta J_{20} = \Gamma(\text{in} m/s^2) \cdot [p\,(\text{in} U.A.)]^4 \cdot \min\left[\frac{10^8}{e^2}; \frac{5.10^8 (1-e)}{\sin i \sin\alpha\, |\cos i|}\right].$$

For β:

$$\delta\left[\beta + \frac{3 J_{20} R^2}{2pm}\left(1 - \frac{3}{2}\sin^2 i\right)\right] = \frac{\frac{2\pi p^3 \Gamma}{mc^2}}{\frac{2\pi aem}{p}}$$

or:

$$\delta\left[\beta + \frac{3 J_{20} R^2}{2pm}\left(1 - \frac{3}{2}\sin^2 i\right)\right] = 1{,}6.10^{+10}\, \Gamma\,(\text{in} m/s^2) \cdot \frac{1 - e^2}{e} \cdot [p\,(\text{in} U.A.)]^3.$$

Example:

$$\text{le} : e = 0{,}5;\ p = 0{,}5\ \text{U.A.};\ i \text{ or } \alpha < 2^{\circ}$$
$$\delta J_{20} = 2{,}5.10^{7} \,.\, \Gamma\ (\text{in } m/s^2)$$
$$\delta\,(\beta + 6\,400\, J_{20}) = 3.10^{9} \,.\, \Gamma\ (\text{in } m/s^2).$$

Remark 1:

For Γ on the order of 10^{-11} to 10^{-13} m/s^2 the accuracy of measurement of L can vary between 3 and 10 meters without changing the accuracy of the measurement of δ and J_{20} (but not the accuracy of γ). This means that $\frac{2\pi p^3 \Gamma}{mc^2}$ varies in effect between several hectometers to several meters.

Remark 2:

One of the principal advantages of using an orbit in the ecliptic plane is that the components of Γ normal to this plane have almost no effect on the measurements and that it is only necessary to counteract the other two.

Remark 3:

A probe in orbit around Venus would be very useful in measuring many characteristics of this planet. However, with respect to β and J_{20} it only gives:

$$L_1 = \beta.64 \text{ m} + J_{20}.280 \text{ km.}$$
$$L_2 = J_{20}.170 \text{ km.}$$

In effect the eccentricity and inclination of the orbit of this planet are very small ($e = 0.0068$; $i = 3.9°$; $\alpha = 3.4°$).

On the other hand, there are a number of complicated perturbations acting on such a satellite. For example, the meteorological variations of the Venus atmosphere (the mass of which is on the order of 4.10^{-4} times the mass of this planet) results in variations in the Venus

potential on the order of 10^{-5}. This results in position perturbations of the satellite on the order of 200 m *per revolution* for a satellite at a distance of 2 radii, and 40 m per revolution for a distance of 10 radii.

For a satellite around Mars, or even better for a probe located on the surface of the planet, (which avoids the majority of the perturbations) the corresponding numbers are:

$a \simeq p = 1{,}52$ U.A.; $e = 0{,}0933$; $i = 5^0{,}6$; $\alpha = 1^0{,}85$;
$L_1 = \beta$ 875 m + J_{20} 1860 km
$L_2 = J_{20}$ 70 km.

The conditions are on the whole better than for Venus, especially if we consider the analysis of the perihelion motion of Mercury which results in a supplementary equation and makes it possible to ignore L_2. On the other hand, the planets result in perfect drag compensation and the accuracy of the measurements increases in proportion with the number of revolutions. All this makes it very difficult to compare the various possibilities, and the following solutions may be considered:

Solution I:

Drag-free solar probe ($\Gamma \underset{\sim}{=} 10^{-11}$ m/s^2) with a laser on board on an eight-month orbit tangent to that of the Earth:

Accuracy of γ: $\underset{\sim}{=}$ 3 to 4.10^{-4}.
Accuracy of β: $\underset{\sim}{=}$ 10 to 20%.
Accuracy of J_{20}: $\underset{\sim}{=}$ 2 to 4.10^{-5}.

Solution II:

Very well drag-compensated solar probe ($\Gamma \underset{\sim}{=} 10^{-2}$ m/s) without a laser and on the same orbit:

Accuracy of γ or β: 1 to 2%.
Accuracy of J_{20}: $\underset{\sim}{=}$ 2 to 4.10^{-6}.

Solution III:

Combination of the two preceding solutions with two different probes. We obtain essentially: $\delta\gamma \underset{\sim}{\sim} 3$ to 4.10^{-4}; $\delta\beta \underset{\sim}{\sim} 1\%$ and $\delta J_{20} \underset{\sim}{\sim} 1$ to 3.10^{-6}.

Solution IV:

Probe in orbit around Mars or even placed on the planet (Project Viking) and used for one or two revolutions.

The accuracy of the results depends considerably on the way the distance of the probe is measured:

Distance measured by a laser up to 3 meters:

$$\delta\gamma \simeq 2.10^{-4}; \delta\beta \simeq 1\ \%; \delta J_{20} \simeq 2.10^{-6}.$$

Distance measured by radio-electric means to 10 or 15 meters (except during superior conjunction where the measurement is worse):

$$\delta\gamma \simeq 1 \text{ à } 2\ \%;\ \delta\beta \simeq 3\ \%;\ \delta J_{20} \simeq 1 \text{ à } 2.10^{-5}.$$

The numbers are much poorer for Venus, even if a larger number of revolutions is used.

It is appropriate to state that the smalless of the "useful lengths" L_1 and L_2 which can be used in the case of planetary probes makes it necessary to analyze the motion of these planets to a high degree of accuracy. It is also necessary to analyze the minor planets; the error in the mass of Mercury can result in deviations on the order of hectometers. These errors must be taken into account with an accuracy of one meter for planetary probes and an accuracy of only 10 meters or even a hectometer (100 meters) for heliocentric probes (conversely, the study will result in a better knowledge of the mass of these perturbing bodies).

These four solutions and the corresponding accuracies take into account the equations resulting from the "relativistic advance" of the perihelion of Mercury. This advance is known up to approximately 1%. However, it does not take into account the one given by the motion of the node of Mercury along the solar equator. This motion leads to $J_{20} = -10^{-6} \pm 2.10^{-6}$ for the J_{20} of the Sun. However, this has been contested by certain scientists, because definition of the Sun's equator has not been adequately defined.

V. CONCLUSION

The coefficients β and γ of the Robertson tensor and the J_{20} of the Sun can be measured by either using one of several heliocentric, drag-free probes (the perturbing, residual, long-period accelerations on the order of 10^{-11} to 10^{-2} m/s^2). This can also be done by placing a probe on Mars and by measuring the distance of these probes to an accuracy of 3 to 15 meters.

The orbital analysis is much more complicated in the second case than in the first case.

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